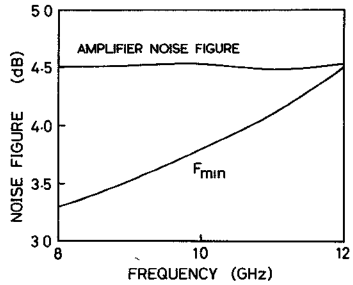

Fig. 8. Minimum noise figure of the transistor and noise figure of the single-stage amplifier.

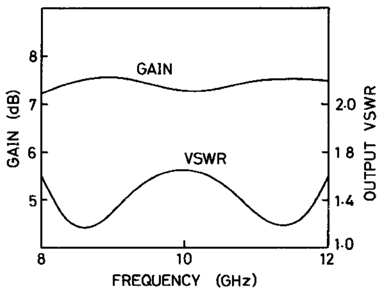

Fig. 9. Gain and output VSWR of single-stage amplifier.

noise figure are plotted against frequency. The amplifier noise figure is close to 4.5 dB across the passband as expected. It is worth noting that the approximated function of Fig. 6 represents a good impedance match between the input coupling network and Y_{of}^* at 12 GHz, but at other frequencies the function falls off in magnitude and the match is degraded. The gain–bandwidth limitations associated with the function are therefore considerably less severe than for a Chebyshev function which approximates a good match across the entire passband. This property of the approximated function permits a lower amplifier noise figure at 12 GHz than is generally available with a Chebyshev coupling network, but the improvement at 12 GHz is achieved at a cost of increased noise figure at lower frequencies.

As well as producing the prescribed noise figure characteristic, the input network affects the available gain of the transistor. For this example, the available gain of the transistor is almost constant across the amplifier passband. The lossless output matching network in Fig. 7 has been designed by a direct synthesis technique [8] using a Chebyshev gain characteristic with 0.25 dB of ripple. The computed amplifier gain and output VSWR is shown in Fig. 9.

IV. CONCLUSION

The loci of constant overall noise figure derived in this short paper can be used to graphically investigate the noise performance of an amplifier for various preamplifier source admittances and main amplifier noise figures. This has been illustrated with an example. An input coupling network synthesis method has been described for noise figure and noise measure specifications. The method enables a wide variety of broad-band design problems to be handled since it reduces the low-noise design problem to an impedance-matching problem which can be solved by well-known analytical techniques. Advantages of the method are related to its simplicity compared with existing low-noise design methods and to the information it can provide regarding noise figure limitations for a given transistor and bandwidth.

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Bias Frequency Modulation of GaAs Millimeter-Wave Diode Oscillators

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Abstract—**A method for estimation of FM bias modulation sensitivity of a Gunn-diode oscillator is presented under the assumption of incomplete domain formation for a very short diode. Experimental measurements at 33 and 11.3 GHz are shown as compared with this estimate and the decrease of sensitivity with external Q, for modulation frequency high enough to eliminate thermal effects, is demonstrated.**

I. INTRODUCTION

Space-charge waves in gallium arsenide (GaAs) have been considered in terms of an elegant analysis [1] by fitting the best experimental measurements of electron velocity v, as a function of electric field E [2], to the expression $1/v = A + BE$, with empirical choice of constants A and B. Experimental results [1] for the rate of change of phase delay θ of the space-charge wave with respect to variation of dc voltage V, show that $\partial\theta/\partial V$ is approximately $B\omega$ for microwave frequency $\omega/2\pi$, until biasing fields exceed 10^6 V/m. For higher fields, velocity saturation causes decrease from $B\omega$.

In this short paper, these effects are related to the phase of the device impedance or admittance under the assumption that the space-charge growth is *small* for short lightly doped samples used in oscillators. A formula is then developed which gives an estimate of the sensitivity of the oscillator to direct frequency modulation (FM) by perturbation of the bias voltage, once the Q of the oscillator is known. Throughout this work, we do not distinguish between loaded Q_L and external Q_{ext}, since the unloaded Q_U was found experimentally to be orders of magnitude higher. Hence we assume $1/Q_L = 1/Q_U +$

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$1/Q_{ext} \doteq 1/Q_{ext}$. For these short devices we have incomplete domain formation—not quenched domains.

II. PHASE PERTURBATIONS IN THE DIODE

The complex problems of discussing the bias FM of Gunn oscillators have been indicated by several authors [3], [4]. It is, of course, very difficult to cover general cases of thermal and other modulation effects even for stable domains. Also, exact analysis is open to question when the lengths of the devices become so short that mature domains are unlikely to form. The devices used in our experiments at ≈ 11.3 GHz have active lengths $l \approx 8$ μm and doping density $n \approx 2 \times 10^{15}$ carriers/cm^3. For the devices at ≈ 33 GHz we have lengths $l \approx 2.5$ μm and doping density $n \approx 4 \times 10^{15}$ carriers/cm^3. Hence domain formation may be incomplete. Previous authors [4] have suggested that the device behavior may lie between the space-charge-free condition and the stable-domain condition, for short lightly doped samples.

In the spirit of these assumptions, we attempt—at least as a first approximation—to estimate the phase delay of the RF space-charge wave. For the study of space-charge waves in GaAs traveling wave *amplifiers*, Robson *et al.* [1] assumed a velocity v versus field E of form

$$\frac{1}{v} = A + BE \tag{1}$$

where

$$A = 2.5 \times 10^{-8}\ \text{s/cm}$$
$$B = 0.64 \times 10^{-11}\ \text{s/V}.$$

This yielded the phase delay of the RF space-charge wave in the form

$$\theta = \omega(Al + BV) \tag{2}$$

where l is the active length, $\omega/2\pi$ the microwave frequency, and V is the bias voltage.

If we take V, now, as our bias plus modulating voltage, and choose a modulating frequency of $\gtrsim 1$ MHz so that we can neglect thermal effects [15] we obtain

$$\frac{\partial\theta}{\partial\omega} = Al + BV \tag{3}$$

and

$$\frac{\partial\theta}{\partial V} = B\omega. \tag{4a}$$

Now examination of [1, fig. 3] shows that $\partial\theta/\partial V$ is $\approx B\omega$ until biasing fields exceed ≈ 700 V/710 μm, i.e., $\approx 10^6$ V/m. At higher fields $\partial\theta/\partial V$ is decreased owing to velocity saturation causing departure from (1).

The devices used for the experiments at ≈ 11.3 GHz were subjected to average biasing fields of ≈ 7 V/8 μm, i.e., 8.75×10^5 V/m. Hence if high-field space-charge domain growth is not severe, we may have a situation between the space-charge-free condition and the stable-domain condition so that (4a) is a reasonable approximation. This assumption appears to give good agreement between experiment and the following theoretical estimate of the FM bias sensitivity of the X-band devices.

For the devices used at ≈ 33 GHz, the average biasing fields were ≈ 4 V/2.5 μm, i.e., 1.6×10^6 V/m. In these cases, $\partial\theta/\partial V$ tends to $\approx \frac{1}{2}B\omega \rightarrow \frac{1}{3}B\omega$. Hence in what follows we shall take, for the devices at 33 GHz:

$$\tfrac{1}{2}B\omega > \frac{\partial\theta}{\partial V} > \tfrac{1}{3}B\omega. \tag{4b}$$

We wish to compare the phase perturbation given by (4a) and (4b) with the phase perturbation of the device impedance or admittance. Only $\partial\theta/\partial V$ needs consideration since we can neglect the term $\partial\theta/\partial\omega$.[1] Moreover, since all theoretical models of bulk semiconductors are questionable [5] and effective domain capacitances require empirical corrections of the order of 50 percent [6], it is only realistic to compare orders of magnitude.

We consider the change in the displacement current due to the small bias voltage perturbation (i.e., modulation) ΔV. Following Copeland [7] we write the change as

$$\Delta\left(\epsilon\frac{\partial E}{\partial t}\right) = -\epsilon\Delta\left(\frac{\partial E}{\partial x}\cdot\frac{\partial x}{\partial t}\right)$$
$$= -\epsilon\Delta\left(\frac{q\tilde{n}}{\epsilon}\cdot\frac{\partial x}{\partial t}\right) = -q\tilde{n}\Delta v$$

where $q\tilde{n}$ is the space-charge density. Hence we can write the phase perturbation (due to the increment of quadrature microwave RF displacement current) as $\approx -\Delta v/v$.

Also, if we consider the domain capacitance current using [6] we have the resistive and capacitive currents from [6, eqs. (46) and (41)] as

$$I_v = env_vA$$

where v_v is the high-field electron velocity and

$$-\frac{dQ}{dt} = A\,(en\epsilon/2V_{ex})^{1/2}\frac{dV_{ex}}{dt}$$

(Q being the charge, *not* the oscillator "Q").

Here we have retained the notation of [6] viz: v_v is the high-field electron velocity, A the area (not to be confused with the parameter A in our velocity field function), and V_{ex} is the domain voltage given from Kurokawa's theory [5] as

$$\frac{dV_{ext}}{dt} = \left(\frac{2neV_{ex}}{\epsilon}\right)^{1/2}(v_0 - v_v)$$

where v_0 is the electron velocity outside the domain. Hence the capacitive current is simply

$$Ane(v_0 - v_v).$$

Thus any phase lag due to perturbation of v_0 will be $\approx -(\Delta v_0/v_v) \approx -(\Delta v/v)$ since Δv is very small while v and v_v have the same order of magnitude.

Assuming small space-charge growth and $v \approx l/\tau = (\omega/2\pi)l$ we have, for bias voltage modulation ΔV

$$\left(-\frac{\Delta v}{v}\right)\Big/\ \Delta V \approx v\Delta(1/v)/\Delta V$$
$$= v\frac{B\Delta E}{\Delta V}$$
$$\approx \frac{v}{l}B = \frac{\omega B}{2\pi}.$$

It is worth noting that displacement current acts as a shunt current, thus supporting the equivalent circuit of [8] which refutes [9], [10], and [11].

Hence, our expressions (4a) and (4b) are of reasonable orders or magnitude for estimates of the phase perturbation of the device impedance or admittance.

While the phase of the device impedance could be derived directly from (1) and the above model including displacement and capacitive current, it is interesting to see that this idealized model gives the same order of magnitude as the phase delay of the RF space-charge wave.

[1]See Section IV.

III. THE OSCILLATION CONDITION

Following Kurokawa [12] we write $Z(\omega)$ as the circuit impedance and $-\bar{Z}$ as the device impedance where the frequency dependence of $-\bar{Z}$ is neglected. This implies that we can neglect $\partial\theta/\partial\omega$ and we can indeed justify this for our problem. If we take the oscillation condition to be

$$[Z(\omega) - \bar{Z}]I = 0 \tag{5}$$

then any voltage modulation must not disturb the condition (5), so that we can postulate an equivalent resonant circuit in accordance with Kurokawa's equations [12, eqs. (2), (11)]

$$2L(\omega_0 - \omega_a) + \bar{X} = 0$$

$$R_a + R_L - \bar{R} = 0 \tag{6}$$

where the overbars refer to the diode reactance and resistance. These conditions imply that

$$d\theta = -d\phi \tag{7}$$

where $d\phi$ is the phase change of the circuit due to frequency change. Condition (7) can be identified with [13, eq. (8)] for the analogous problem of electron beam devices.

IV. RESONATOR PHASE PERTURBATION AND FM SENSITIVITY OF THE SYSTEM

Approximating ϕ as given for a simple resonant circuit we write

$$\phi = \arctan Q\left(\frac{\omega}{\omega_a} - \frac{\omega_a}{\omega}\right) \tag{8}$$

where ω_a is the resonant frequency of the circuit.

Following [13]

$$\frac{d\omega}{dV} = \frac{-(\partial\theta/\partial V)}{d\phi/d\omega + \partial\theta/\partial\omega}$$

$$= \frac{-(\partial\theta/\partial V)}{d\phi/d\omega + (Al + BV)}$$

from (3). From (8) we have

$$\frac{d\phi}{d\omega} = \frac{1}{1 + Q^2[\omega/\omega_a - \omega_a/\omega]^2} \times Q\left(\frac{1}{\omega_a} + \frac{\omega_a}{\omega^2}\right)$$

i.e.,

$$\frac{d\phi}{d\omega} \doteq \frac{2Q}{\omega} \tag{9}$$

since $(\omega - \omega_a)/2\pi$ is a few megahertz while $\omega/2\pi \approx 11.3$ or 33 GHz. Even for Q as low as 75 we would have $2Q/\omega \approx 7.5 \times 10^{-10}$ for 33 GHz or $\approx 22.5 \times 10^{-10}$ for X band. Now, for 33 GHz we have $l \approx 2.5$ µm and $V \approx 4$ V while for X band we have $l \approx 8$ µm and $V \approx 7$ V.

It is readily verified that under these conditions the quantity $\partial\theta/\partial\omega = (Al + BV)$ can be neglected, but at higher voltages a decrease in FM sensitivity can indeed be observed in practice.

Neglecting $\partial\theta/\partial\omega$, and using (9) we have

$$\frac{d\omega}{dV} = \frac{-(\partial\theta/\partial V)}{d\phi/d\omega} = \frac{-(\partial\theta/\partial V)}{2Q/\omega}. \tag{10}$$

From (10) and (4a) or (4b) we have, respectively,

$$\text{for } X\text{-band samples:} \quad \left|\frac{d\omega}{dV}\right| = \frac{\omega^2 B}{2Q} \tag{11a}$$

or

$$\text{for 33-GHz samples:} \quad \frac{\omega^2 B}{4Q} > \left|\frac{d\omega}{dV}\right| > \frac{\omega^2 B}{6Q}. \tag{11b}$$

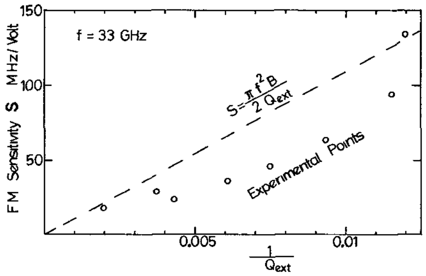

Fig. 1. Comparison of formula for velocity approaching saturation with measurements at 33 GHz.

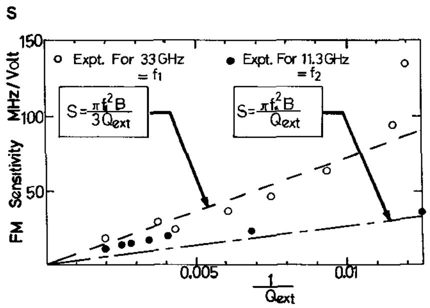

Fig. 2. Upper graph: Comparison of formula for velocity closer to saturation with measurements at 33 GHz. Lower graph: Comparison of formula for negligible saturation of velocity with measurements at 11.3 GHz.

It is worth emphasizing that even X-band devices would require (11b) if the fields were higher.

V. EXPERIMENTAL COMPARISONS

Defining the FM sensitivity as

$$S = \left|\frac{df}{dV}\right|$$

we have

$$S = \frac{\pi f^2 B}{Q} \tag{12a}$$

for X band, and

$$\frac{\pi f^2 B}{2Q} > S > \frac{\pi f^2 B}{3Q} \tag{12b}$$

for $\approx$ 33 GHz. In all cases, the Q used is in fact the external Q as indicated in Figs. 1 and 2. Fig. 1 shows that most of the experimental points do in fact fall below the line $S = \pi f^2 B/2Q$ for 33 GHz. For $Q < 80$, higher order terms appear necessary. Fig. 2 shows that most of the experimental points for 33 GHz fall near the line $S = \pi f^2 B/3Q$ although some do fall slightly below the line. For the X-band devices at $\approx$ 11.3 GHz there is, of course, less ambiguity and the line $S = \pi f^2 B/Q$ yields good estimates which are only slightly less than the experimental values.

The method used to measure the external Q was adopted from [14]. The diodes were waveguide mounted with usual plunger tuning. The bias modulation voltages were applied quite simply through a capacitor and monitored at the diode contact. The FM deviation was measured by the Bessel function zero method of observing the carrier nulls.

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Variation of the Electrical Characteristics of an Inhomogeneous Microstrip Line with the Dielectric Constant of the Substrate and with the Geometrical Dimensions

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***Abstract*—Studying systematically the variations of electrical characteristics of microstrip lines with the width w of the line, the thickness h, and the dielectric constant ϵ_r of the substrate, we have obtained a perfect linear variation with ϵ_r. Then using a least squares method, we have been able to give an analytical expression of capacitances usable for $1 \leq \epsilon_r \leq 100$ and $0.04 \leq w/h \leq 10$. The importance of this result is that we can give impedances and phase velocities without any computation.**

It is well known that all the parameters of microstrip lines or couplers can be calculated from the elements of the matrix of capacitances (S) for a purely dielectric substrate [1]. The matrix of the inductances (M) can be deduced from the last one by the relation

$$(M) = \frac{1}{c^2} (S_0)^{-1}$$

where c and S_0 are, respectively, the velocity of light and the matrix (S) in the vacuum.

Up to now, in the case of the quasi-TEM approximation the capacitances can be computed by different numerical methods [2]–[4]. These methods are lengthy and not very easy to use. We have used an accelerated finite differences method [5] which is more convenient (and also universal) to calculate these capacitances, but we have thought that it would be very interesting for practical uses to derive a formula giving directly the capacitances of the microstrip lines and couplers only in terms of the geometrical parameters and the dielectric substrate. Knowing the capacitances, we can obtain impedances, effective dielectric constant, coupling coefficient, etc.

Another interesting point is that such a formula will be able to permit users to define the geometrical dimensions of the lines to obtain a fixed value of the capacitance for a given dielectric substrate.

First, we have studied the single inhomogeneous microstrip line. Fixing w/h, we have calculated the capacitance for several values of ϵ_r between 1 and 100. Plotting the capacitances in terms of ϵ_r, we have obtained a straight line for any value of w/h (Fig. 1). The slope p of these straight lines increases with w/h.

We can write $C = p(\epsilon_r - 1) + C_0$, where p depends only on w/h. After this, we have plotted p in terms of w/h with the purpose of obtaining a polynomial approximation of this function, the graph of which would cross over at the calculated points. We have obtained a curve having an oblique asymptotic direction for the large values of w/h (Fig. 2).

[For the small values of w/h, we have taken $h = 50$ instead of $h = 8$ to improve the precision of the computation (Fig. 3).]

This is physically correct because when w/h becomes very large, the capacitance of the line tends to the one of a perfect plane capacitor

$$C = \epsilon_0 \epsilon_r \frac{w}{h} = \epsilon_0 \frac{w}{h} (\epsilon_r - 1) + \frac{\epsilon_0 w}{h}.$$

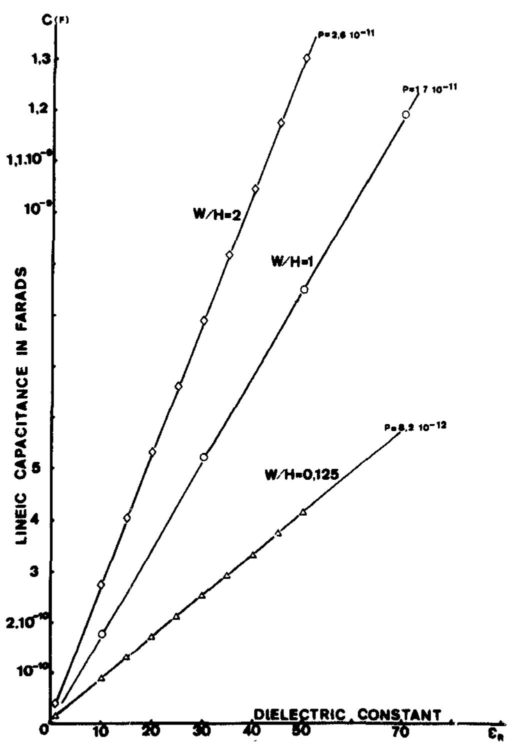

Fig. 1. Variation of the linear capacitance of the line with the dielectric constant of the substrate.

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